Discussion 9 Worksheet
Tangent planes (revisited) and optimization

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MATH 53 Multivariable Calculus

1 Tangent Plane

Find the equation of the tangent plane.

(a) $2(x-2)^2 + (y-1)^2 + (z-3)^2 = 10$ at $(3, 3, 5)$;

(b) $xy^2z^3 = 8$ at $(2, 2, 1)$;

(c) $x + y + z = e^{xyz}$ at $(0, 0, 1)$.

(d) Show that the equation of the tangenet plane to the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ at the point (x_0, y_0, z_0) can be written as

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = 1.$$

(e) Show that the sum of the $x-$, $y-$, and $z-$intercepts of any tangent plane to the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{c}$ is a constant.

2 Maxima and Minima

Find the local maximum and minimum values and saddle point(s) of the function.

(a) $f(x, y) = x^2 + y^4 + 2xy$

(b) $f(x, y) = xy + e^{-xy}$

3 Challenge

Suppose that the direction derivatives of $f(x, y)$ are known at a given point in two nonparallel directions given by unit vectors $\vec{u}$ and $\vec{v}$. Is it possible to find ∇f at this point? If so, how would you do it?

4 True/False

(a) T F A point that makes $\nabla f = \vec{0}$ corresponds to a critical point.

(b) T F If the second derivative test fails, it is impossible to say anything about the critical point in regards to it being a maxima or minima.

Note: These problems are taken from the worksheets for Math 53 in the Spring of 2021 with Prof. Stankova.